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STABILITY OF THERMAL CONVECTION IN A SALINITY GRADIENT DUE TO LATERAL HEATING

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Abstract—Experimental investigations have been conducted to study the behavior of the flow field induced by lateral heating in a stably stratified fluid of constant gradient. The experiments were carried out in a tank $5 \times 7\frac{7}{8} \times 11\frac{3}{4}$ in. high; two opposite walls were made of aluminum and kept at different but constant temperatures, while the other two were of plexiglass for flow observation. The governing parameter is shown to be the Rayleigh number based on the length scale which is the potential rise of a heated fluid element in a stably stratified surrounding. At subcritical Rayleigh numbers, the fluid rises, almost imperceptibly, parallel to the heated plate and the lateral transport of heat is entirely by conduction. At supercritical Rayleigh numbers, the flow becomes unstable and cellular convection results, which is clearly visible in a shadowgraph. The cells are of approximately equal size and they appear simultaneously all along the heated wall. They grow in the lateral direction and finally reach the opposite wall. The critical Rayleigh number above which cellular convection occurs has been experimentally determined to be 15000 ± 2500.

NOMENCLATURE

- g, gravitational acceleration [ft/s^2];
- h, reference length $= \alpha \Delta T/\phi_0$ [ft];
- p', pressure [lb/ft^2];
- R, Rayleigh number $= g\alpha\Delta T h^3/\kappa\nu$;
- S', salinity [wt %];
- S, nondimensional salinity $= -(S' - S'_0)/h(dS'/dz')_0$;
- t', time [s];
- t, nondimensional time $= t'\kappa/h^2$;
- T', temperature [°F];
- T, nondimensional temperature $= (T' - T'_0)/\Delta T$;
- ΔT, temperature difference [°F];
- $\boldsymbol{v}'$, velocity [ft/s];
- $\boldsymbol{v}$, nondimensional velocity $= \boldsymbol{v}'h/\kappa$;
- x', distance normal to heated wall [ft];
- x, $= x'/h$;
- z', distance along heated wall [ft];
- z, $= z'/h$.

Greek symbols

- α, coefficient of volume expansion [°F^{-1}];
- β, $= \dfrac{1}{\rho'}\left(\dfrac{\partial \rho'}{\partial S'}\right)_{p', T'}$ [wt %$^{-1}$];
- κ, thermal diffusivity [ft^2/s];
- κ_s, salt diffusivity [ft^2/s];
- ν, kinematic viscosity [ft^2/s];
- ρ', density [slugs/ft^3];
- σ, Prandtl number $= \nu/\kappa$;
- ϕ_0, initial density gradient $= -\beta(dS'/dz')_0$ [ft^{-1}];
- $\boldsymbol{\Omega}'$, vorticity [s^{-1}];
- ω', $\boldsymbol{j}.\boldsymbol{\Omega}'$;
- ω, nondimensional vorticity $= \omega' h^2/\kappa$.

Subscript 0 refers to reference state.
Primes indicate dimensional quantities.

1. INTRODUCTION

IN THE late 1800's, Brewer [1] and Barus [2] reported the curious phenomenon that under certain circumstances the subsidence of fine particles in water took place in stratified layers. Both investigators studied the effect of the type and number of particles and of different aqueous solutions on the appearance and evolution of these layers. Neither of them was able to arrive at a satisfactorv explanation of the phenomenon. In 1923 Mendenhall and Mason [3], after exhaustive experimental investigation, concluded that "stratification only occurs when there is a lateral temperature gradient across the liquid". Their explanation is as follows: Due to the normal sedimentation process, there is a positive gradient downwards in the number density of the particles, thus the mean density of the fluid is stably stratified. When a lateral temperature gradient is imposed say by cooling of one wall, the cooled fluid element at the top of the container near the cold wall sinks to a level where it no longer experiences any buoyancy, and due to the momentum of the downward flow, it turns away from the wall and into the fluid. The first layer is created. The second layer will be generated in a similar manner; the layers will grow in succession. However, our experiments as well as those of Thorpe *et al.* [4]* in stably stratified salt solution show that under certain (supercritical) conditions, the convection cells or rolls spring up simultaneously along the heated (or cooled) wall. This clearly indicates a stability phenomenon. Thorpe *et al.* [4] have made a linear stability analysis for the case of a stably stratified fluid confined within two parallel plates sustaining horizontal linear gradients in temperature and salinity such that the horizontal density gradient is zero. For a vertical slot at the onset of instability, their solution yields a relationship between the thermal Rayleigh number based on the lateral temperature gradient and the solute Rayleigh number based on the initial vertical salinity gradient. Their theoretically predicted stability limit has been qualitatively confirmed by their experimental results in slots of 3·5 and 6 mm widths.

There has been some recent evidence that layered structures both in salinity and temperature exist in some parts of the ocean and in salt-water lakes. Swallow and Crease [5] have found water with abnormally high temperature and salinity in a depression at the bottom of the Red Sea. This body of water is separated from the normal sea water by sharp temperature and density gradients. Hoare [6] has found layered structure both in temperature and density in an Antarctic lake. It is reasonable to expect that in both cases heating comes from both the bottom and sides. From the experiments of Turner and Stommel [7, 8], it is known that when a stably stratified fluid is heated from below, a layered convection pattern will be generated successively from the bottom up. These layers will ultimately disappear due to continuous agitation of the inherently unstable bottom layer. However, the cells generated by a lateral temperature gradient as observed by us and by Thorpe *et al.* [4] persist for a relatively longer time. A basic understanding of the convection phenomenon in a stably stratified fluid with a lateral temperature gradient may shed some light on the origin and the time evolution of such layers found in oceans and lakes.

Our investigation is concerned with the initiation of instability, which manifests itself in convection rolls or cells, along a heated wall in a tank with a wide gap such that at the time of onset of instability, the presence of the opposite wall is not known by the convecting fluid. We shall show in Section 2 that under this circumstance, the important parameter is the Rayleigh number based on the length scale which is the height of rise of a heated fluid particle in stably stratified surroundings. The experimental apparatus and procedure for the determination of the critical Rayleigh number are presented in Section 3, and the results and discussion follow in Section 4.

2. DIMENSIONAL ANALYSIS

Consider a semi-infinite fluid stably stratified in the vertical direction which is being heated by the boundary wall. Let the cartesian coordinate system (x', z') be such that the wall is at $x' = 0$ and the fluid at $x' > 0$ with z' positive upwards. The initial salinity gradient is $(dS'/dz')_0$. Let the temperature of this fluid be initially at T'_0 and that of the wall $T'_0 + \Delta T'$. The density of this incompressible fluid may be approximated by

* This reference came to our attention when this manuscript was under preparation. Many of the interesting phenomena observed by us have been observed and excellently reported by them.

$$\rho' = \rho'_0 [1 - \alpha(T' - T_0) + \beta(S' - S'_0)] \quad (1)$$

where

$$\alpha = -\frac{1}{\rho'}\left(\frac{\partial \rho'}{\partial T'}\right)_{p',S'}, \qquad \beta = \frac{1}{\rho'}\left(\frac{\partial \rho'}{\partial S'}\right)_{p',T'} \quad (2)$$

and the subscript 0 denotes the reference state. Using the Boussinesq approximation, the equation of motion is

$$\frac{D}{Dt}\boldsymbol{v}' = -\frac{1}{\rho'_0}\nabla p' + \boldsymbol{k}g[\alpha(T' - T'_0) - \beta(S' - S'_0)] + \nu\nabla^2\boldsymbol{v}'. \quad (3)$$

The equation governing the vorticity $\boldsymbol{\Omega}' = \boldsymbol{j}\omega'$ may be obtained from the above by eliminating the pressure:

$$\frac{D}{Dt'}\omega' = \nu\nabla^2\omega' - g\left[\alpha\frac{\partial T'}{\partial x'} - \beta\frac{\partial S'}{\partial x'}\right]. \quad (4)$$

Since both $(\partial T'/\partial x')$ and $(\partial S'/\partial x')$ are negative, it is seen that the horizontal temperature gradient is a source whereas the horizontal salinity gradient is a sink in the vorticity generation term. The relative magnitude of these two terms will determine the behavior of the flow. The continuity, energy and salt diffusion equations are

$$\nabla \cdot \boldsymbol{v}' = 0 \quad (5)$$

$$\frac{DT'}{Dt} = \kappa\nabla^2 T' \quad (6)$$

$$\frac{DS'}{Dt} = \kappa_s\nabla^2 S'. \quad (7)$$

We now introduce a reference length scale h yet to be specified and the following nondimensional quantities which are unprimed:

$$x = x'/h, \quad T = (T' - T'_0)/\Delta T,$$
$$S = -(S' - S'_0)/h(\mathrm{d}S'/\mathrm{d}z')_0, \quad t = t'\kappa/h^2$$
$$\boldsymbol{v} = \boldsymbol{v}'h/\kappa, \quad \omega = \omega' h^2/\kappa.$$

When these quantities are substituted into the vorticity equation we have

$$\frac{1}{\sigma}\frac{D\omega}{Dt} = \nabla^2\omega - \frac{gh^3}{\nu\kappa}\left[\alpha\Delta T\frac{\partial T}{\partial x} - h\phi_0\frac{\partial S}{\partial x}\right] \quad (8)$$

where $\sigma = \nu/\kappa$ is the Prandtl number, and $-\phi_0$ is the initial density gradient

$$\phi_0 = -\beta(\mathrm{d}S'/\mathrm{d}z')_0 = -\frac{1}{\rho'}\left(\frac{\partial \rho'}{\partial z'}\right)_0. \quad (9)$$

If we now choose

$$h = \frac{\alpha\Delta T}{\phi_0} \quad (10)$$

which is the height to which a heated fluid element would rise in an initial density gradient $-\phi_0$, we have for the vorticity equation

$$\frac{1}{\sigma}\frac{D\omega}{Dt} = \nabla^2\omega - R\left(\frac{\partial T}{\partial x} - \frac{\partial S}{\partial x}\right) \quad (11)$$

where the Rayleigh number is defined in terms of $h(10)$

$$R = \frac{g\alpha\Delta T}{\nu\kappa}\left(\frac{\alpha\Delta T}{\phi_0}\right)^3. \quad (12)$$

With this choice of the reference length scale, the lateral gradients in temperature and salinity assume equal importance and the Rayleigh number serves as a measure of the vorticity generation term. The continuity, energy and the salt diffusion equations become

$$\nabla \cdot \boldsymbol{v} = 0, \quad \frac{DT}{Dt} = \nabla^2 T, \quad \frac{DS}{Dt} = \frac{\kappa_s}{\kappa}\nabla^2 S. \quad (13)$$

For salt solutions, the ratio of diffusities κ_s/κ is of order 10^{-2}. In our experimental program, we seek to find the critical Rayleigh number above which convection rolls begin to appear.

3. APPARATUS AND PROCEDURE

Apparatus

A schematic drawing of the experimental apparatus is shown in Fig. 1. The tank in which the experiments were carried out is $11\frac{3}{4}$ in. high × $7\frac{7}{8}$ in. deep × 5 in. wide with two aluminum ($11\frac{3}{4}$ in. × $7\frac{7}{8}$ in.) and two Plexiglas side walls. The aluminum side walls, whose backs are

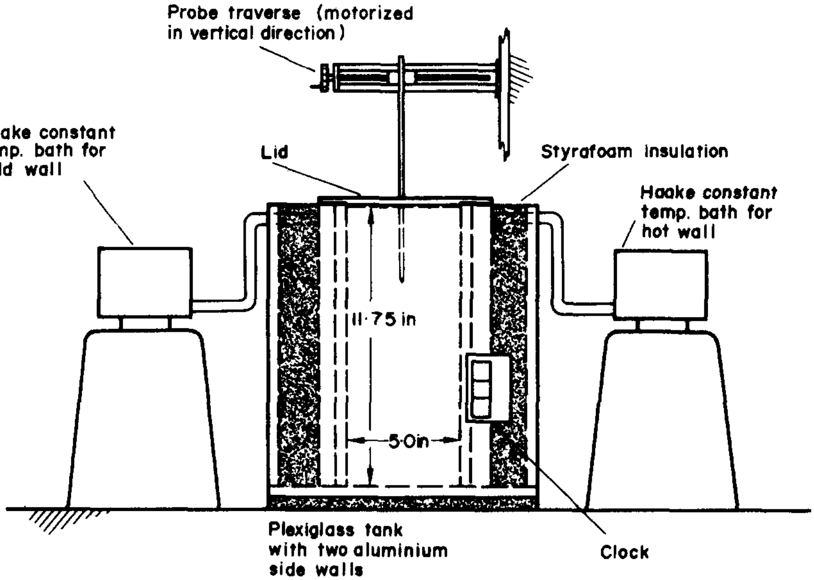

FIG. 1. Sketch of experimental apparatus.

insulated with styrofoam insulation, are provided with counter-flow passages in which water of pre-set temperature may be circulated. Two Haake constant temperature baths with circulating pumps are used to maintain the two aluminum walls at approximately isothermal conditions. Thirteen thermocouples are installed on the surface of each isothermal wall, eleven of which are installed along the vertical and horizontal centerlines of the plate. The remaining two are installed at the two corners. The outputs of all seven thermocouples along the vertical centerlines of both the hot and cold plates are recorded on a Honeywell Recorder along with the water inlet and outlet temperatures. All thermocouples may be read on a Leeds–Northrup potentiometer through a switching device. A Plexiglas lid is put on top of the tank to provide a solid boundary, and it is insulated on top by styrofoam.

A single-electrode conductivity probe was used to measure the local value of salinity. It is constructed of 0·004 in. dia. platinum wire embedded in the drawn-out tip of a small diameter glass tube. This type of probe has been used by Lamb *et al.* [9] and Gibson and Schwartz [10]. Prior to each use the probe tip is coated with platinum black by electroplating with Leeds–Northrup platinizing solution. A $\frac{1}{32}$ in. thick stainless steel plate is placed on the bottom of the tank to serve as a second electrode. The output of the single-electrode conductivity probe is monitored by a Leeds–Northrup conductivity bridge.

A traverse mechanism which is motorized in the vertical direction is mounted above the tank. The vertical position of the probe can be calibrated in the number of turns of a d.c. potentiometer. The single-electrode conductivity probe and a thermocouple probe can be mounted simultaneously on the traverse mechanism. The temperature and salinity at any vertical level may be determined at the same time.

Many methods of flow visualization have been tried. These include: aluminum particles and pearl essence for streak line observation, dye trace and shadowgraph. Each of these methods is suited for examining particular aspects of the flow. We have relied on the shadowgraph method for the bulk of our experimental data because of its ease of operation and at the same time it offers an overall view of the motion in the entire tank. We shall, however, present a streak photograph obtained by using aluminum particles to illustrate the motion in the well developed convection cells. For the shadowgraph, a mercury arc light is placed approximately 30 ft away from the apparatus. The light source passes through a lens which limits the angle of divergence to about 1°. A frosted glass was placed against the Plexiglas wall to make visible the shadowgraph which is in turn photographed by using a 35 mm camera.

Procedure

Salt solutions in distilled water of successively smaller salinity were admitted into the tank in one-in. layers. After the tank was filled, the fluid was left standing for 24 h so that the sharp gradients may smooth out by diffusion and attain a linear salinity gradient. We have programmed the salt diffusion equation for a 12-layer system in finite difference form using an implicit scheme. The calculated results using a $\kappa_s = 1{\cdot}36 \times 10^{-8}$ ft²/s and those measured with the single-electrode conductivity probe after 24 h are shown in Fig. 2. It is seen that the actual diffusion proceeds faster than theory indicates. This is to be expected since there is bound to be some turbulent mixing at the interface of two adjacent layers while the fluid is being admitted.

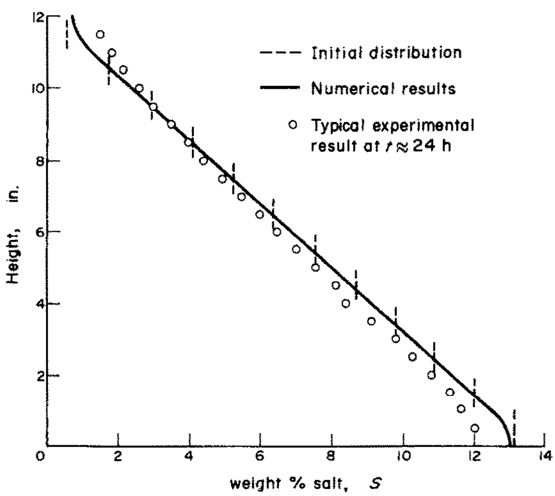

FIG. 2. Salinity distribution.

After the linear salinity profile had been established, the heaters in the constant temperature baths were turned on. The reservoir for the hot wall was set at predetermined temperature and that for the cold wall was set at the ambient temperature (or equivalently the initial temperature of the fluid in the tank). When the water temperature in the hot bath attained the desired level, the pumps were activated thus commencing a test run.

The rise curve of the temperature at the center of the heated wall has a time constant of approximately 3 min. The normalized temperature rise

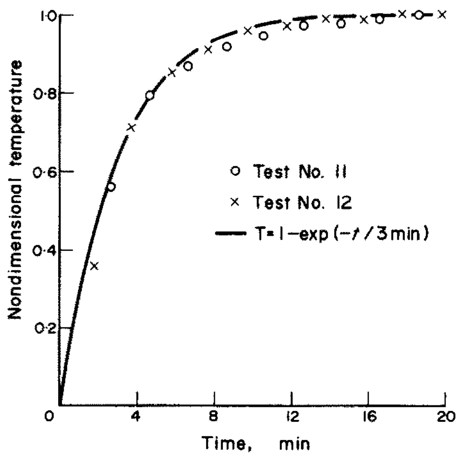

FIG. 3. Temperature rise of the heated wall.

data for two typical test runs (Nos. 11 and 12) are shown in Fig. 3 together with the curve

$$T = 1 - \exp(-t'/3\,\text{min}).$$

The differences are seen to be small. There is a slight temperature non-uniformity in the heated wall on the order of 5 per cent of the temperature difference ΔT in all tests.

4. RESULTS AND DISCUSSION

Preliminary investigation

As a preliminary investigation, we conducted an experiment with a comparatively large temperature difference. Results of this test established firmly that layered convection was indeed a stability phenomenon, and that the convection cells once fully developed persisted for a long time.

The experiment was carried out in an initial density gradient of $-0{\cdot}06$ ft^{-1}. The hot wall was set at a temperature 18·6°F above and the cold wall at 5·9°F below the temperature of the fluid in the tank with an overall ΔT of 24·5°F. The Rayleigh number based on the hot wall temperature difference is 7×10^5. During this experiment, shadowgraph pictures were made by exposing photographic paper directly at the Plexiglas wall. In the shadowgraphs as shown in Fig. 4, the hot wall appears on the left and cold wall appears on the right. At $t = 2$ min, there is a row of approximately evenly spaced convective cells on each isothermal wall, the cells being larger at the hot wall where the local ΔT is about three times that at the cold wall. It is also noted that along the hot wall, the motion of the fluid is noticeably more vigorous than that near the cold wall. The cells at the hot wall grow laterally and at $t = 11$ min these cells have reached the cold wall. Both the top and the bottom cells are larger than the cells in the main body of the fluid. This may be explained by the smaller gradients of salinity near the boundaries. Excluding the top and bottom ones, there are 16 cells at $t = 11$ min. The bottom wall is made of Plexiglas and no special precaution was taken to insulate it from the hot wall. Thus the bottom cell is subjected to heating from the bottom near the hot wall and cooling from the bottom near the cold wall. These added heat sources and sinks make the convection current more vigorous and finally it breaks down the barrier between the bottom cell and the one immediately above. In the top-most cell, the fluid is being cooled both from the side and from the top near the cold wall. Circulation is also increased, and it gradually merges with the one next to it. At $t = 125$ min there are only 13 cells left. Besides losing the ones near the top and the bottom, the 7th one from the top ($t = 11$ min picture) has also merged with its neighbor. At $t = 24$ h there are 5 cells left in the central portion of the tank due to erosion from the top and the bottom. The average cell spacing is approximately the same as that shown in the picture at $t = 125$ min. A plumb line was fixed midway between the two isothermal walls to serve as a reference for determining the slope of the cells. This line can be seen in all the photographs.

The motion of the fluid in the fully developed convection cells was examined using the particle streak photograph. Fine aluminum powders were put in suspension in the fluid prior to the start of the experiment, and they are illuminated from the top by a slit light source approximately $\frac{1}{4}$ in. wide. A typical photograph is shown in Fig. 5 with the hot wall now on the right. One can discern the general counterclockwise circulation in the cell with comparatively quiescent region in the middle of the cell and high shear regions at the cell boundaries.

Determination of the critical Rayleigh number

A series of 15 tests have been carried out to determine the critical Rayleigh number. Our original plan was to start at the low Rayleigh number and try to approach the critical value. However, except for the first three tests in which the Rayleigh numbers were quite a bit below the critical, the scheme did not work as planned. Since the Rayleigh number depends on the third power of the length scale $h = \alpha\Delta T/\phi_0$, and the control over the initial density gradient could not be very precise, the result is that the Rayleigh numbers of the tests leap-frogged back and forth about the critical.

We define the supercritical flow as the state in which convection cells occur along the entire length of the heated wall when the wall temperature has attained its "steady-state" value. This time is about 20 min as shown in Fig. 3. When the Rayleigh number is much larger than the critical, the simultaneous occurrence of the cells takes place on the order of a few minutes; the higher the Rayleigh number, the sooner the instability will occur. By the use of a motion picture camera one could determine the critical

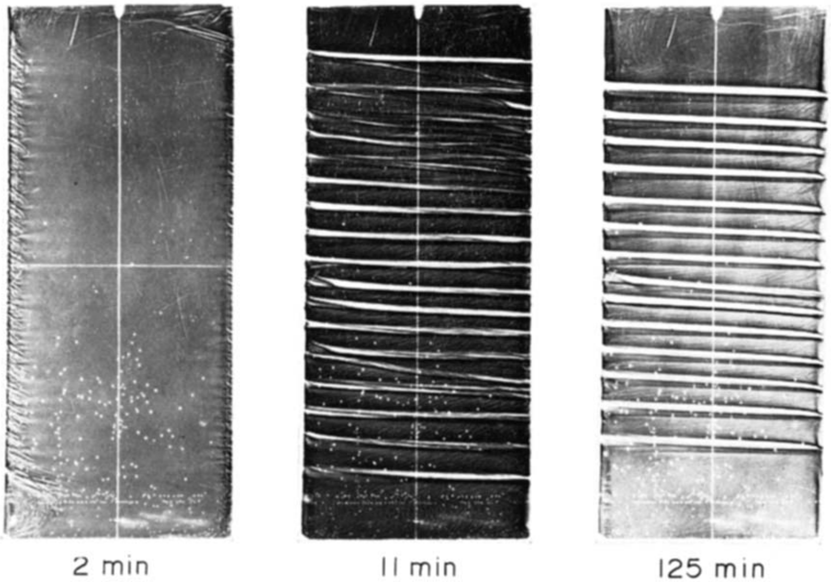

FIG. 4. Layered convection at supercritical Rayleigh number $= 7 \times 10^5$.

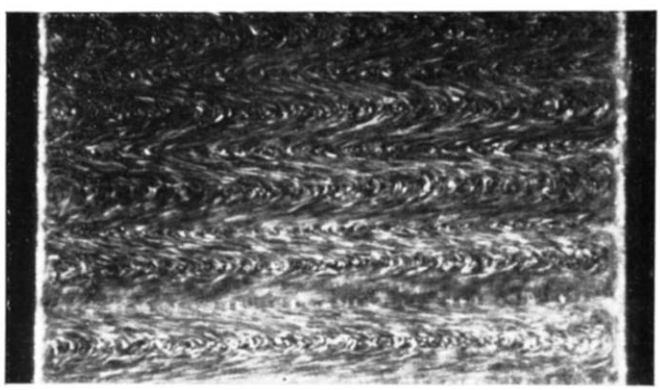

FIG. 5. Convection motion in fully developed cells.

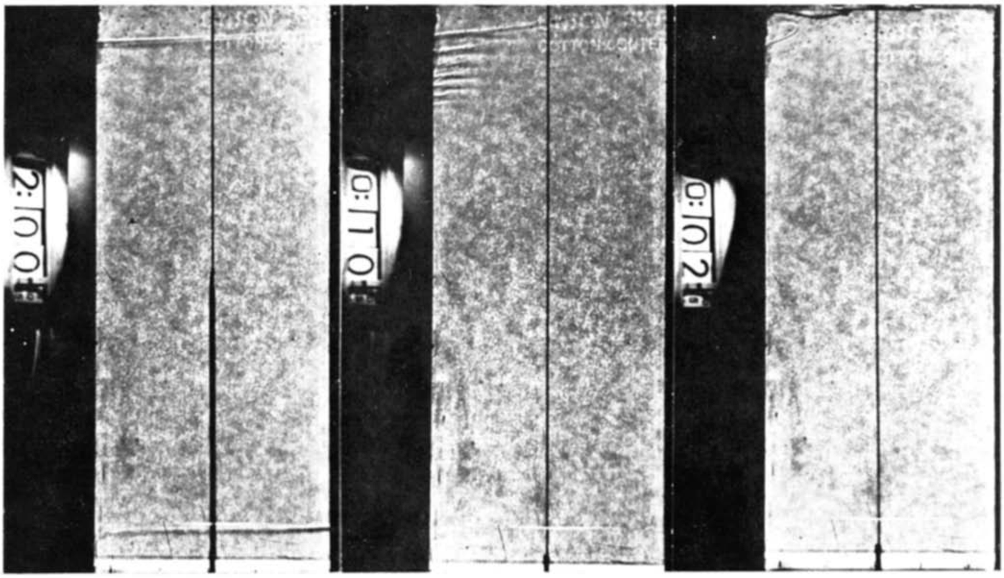

FIG. 6. Subcritical convection regime, $R = 4500$.

a

b

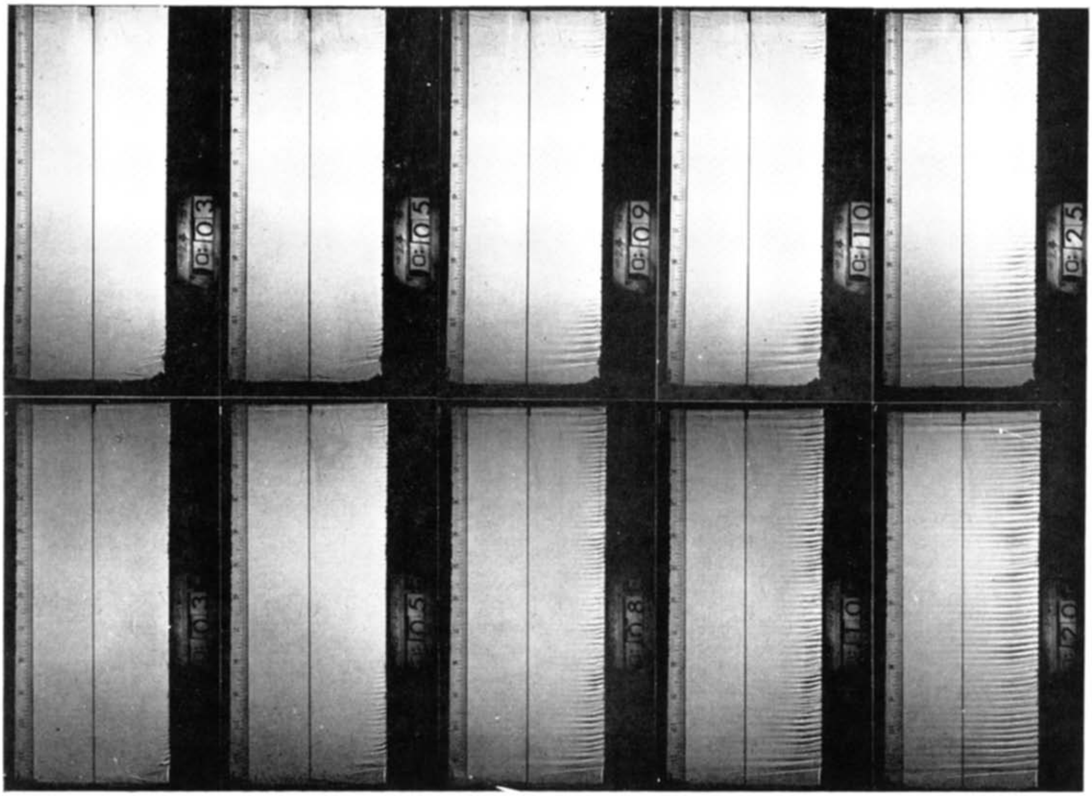

FIG. 8. Transitional and supercritical convection regimes
(a) $R = 15\,700$ (b) $R = 25\,500$.

Table 1. Summary of experimental results

Test no	Rayleigh no. $R \times 10^{-4}$	Cell size $h/(\alpha\Delta T/\phi_0)$	ϕ_0 (ft^{-1})	ΔT (°F)	T_0 (°F)	Mean S (wt %)	$\kappa \times 10^6$ (ft^2/s)	$\nu \times 10^5$ (ft^2/s)	$\alpha \times 10^4$ ($°F^{-1}$)
1	0·08	*	0·0600	3·20	75·5	3·46	1·58	0·99	1·67
2	0·09	*	0·0558	3·00	77·0	4·52	1·59	0·99	1·74
3	0·45	*	0·0617	5·34	70·7	4·49	1·58	1·05	1·61
8	1·13	*	0·0521	5·20	79·4	4·06	1·60	0·95	1·79
6	1·42	0·907	0·0580	6·18	76·2	4·53	1·59	0·99	1·75
5	1·52	*	0·0567	6·32	74·7	4·52	1·59	1·00	1·71
9	1·57	*	0·0525	6·07	74·8	4·19	1·59	1·00	1·69
11	1·81	0·756	0·0527	5·97	78·4	4·14	1·60	0·96	1·77
14	2·44	0·837	0·0927	9·64	74·6	6·71	1·60	1·02	1·84
13	2·54	0·835	0·0788	8·23	78·4	6·46	1·61	0·98	1·91
12	2·55	0·847	0·0886	9·08	77·7	6·46	1·61	0·99	1·90
10	2·68	0·689	0·0413	5·44	80·0	3·74	1·60	0·94	1·78
7	4·40	0·747	0·0479	7·62	72·9	4·09	1·58	1·02	1·64
4	5·05	0·674	0·0587	9·20	72·0	4·44	1·58	1·04	1·64
15	5·39	0·973	0·0960	11·91	74·4	7·25	1·61	1·03	1·87

* Denotes subcritical convection regime.

time when the cells appear for different supercritical Rayleigh numbers as is done by Kirchner and Chen [11] for the time dependent Couette flow. What we are determining is the lower bound of the critical Rayleigh number below which no simultaneous occurrence of convection cells may take place.

A summary of experimental results is presented in Table 1. The tests are arranged in increasing Rayleigh number, which ranged from about 800 to 54000. The test numbers are in chronological order. The values of the initial density gradient ϕ_0, the "steady-state" ΔT, and the salinity S are taken at the mid-height of the tank. The property values which are functions of both salinity and temperature are obtained by using [12].

For each of the tests, shadowgraph pictures are taken every minute after the start of the test to about 10 min into the experiment. Then pictures are taken at 5 min or longer intervals depending upon the state of the flow. For supercritical flows, the cell sizes are determined by averaging over the tank height at $t = 20$ min. It can be seen from Table 1 that the ratio of the actual cell size to the length scale h ranges from 67 to 97 per cent with an average of 80 per cent. This indicates that the length scale h is a reasonable choice.

The subcritical convection regime is illustrated by three shadowgraphs taken from test No. 3 with $R = 4500$ as shown in Fig. 6. At $t = 2$ min which is indicated on the timer, there is a starting cell at the lower corner of the hot wall. This is due to the smaller density gradient and the presence of the lower boundary. As soon as the fluid near the wall rises due to buoyancy, there is horizontal motion along the bottom wall which tends to induce the recirculating flow of the fluid which has reached its own density level. At $t = 10$ min, there are 6 cells above the bottom cell, and they are developed successively in the manner described by Mendenhall and Mason [3]. The successive growth of the layers is clearly exhibited by the progressively shorter lengths of the upper layers. At $t = 2$ h there are only two well established cells, one near the top and one near the bottom wall. A temperature traverse made at $t = 1·75$ h is shown in Fig. 7 together with the transient heat conduction solution using $\kappa = 1·58 \times 10^{-6}$ ft^2/s. It shows that the dominant mode of heat transfer is by conduction. The slow upward motion of the fluid in this subcritical

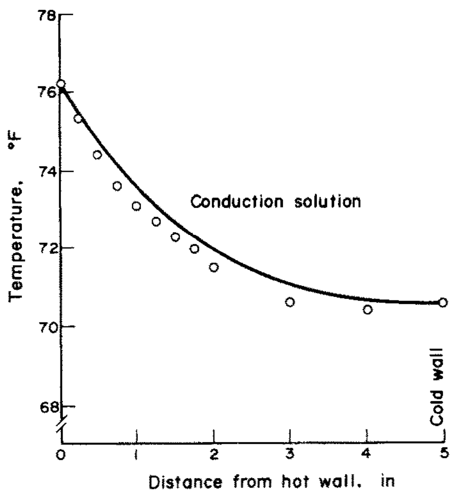

FIG. 7. Temperature distribution in subcritical convection regime. At R = 4500 and t = 1·75 h conduction solution is for = $1{\cdot}58 \times 10^{-6}$ ft²/s.

convection regime does not contribute to the transfer of heat.

For a comparison of the transitional and supercritical convection regimes, shadowgraphs at comparable times for test Nos. 9 and 12 are shown in Fig. 8. Test No. 9 with R = 15700 is at the upper limit of the subcritical regime. As shown in Fig. 8a, the layered convection pattern is due entirely to the effects discussed earlier. At t = 25 min the growth has reached about 5 in. from the bottom. Between the 5–10 in. levels, the enclosure is free of cells. At a supercritical Rayleigh number R = 25500, Fig. 8b, the initial motion at t = 3 min is induced by the end walls. However, at t = 5 min, the flow has become unstable and the cellular motion throughout the tank becomes evident. The evolution of the cellular convective motion is recorded in the next three pictures. At t = 20 min the cells almost reached the opposite wall.

From our data, the critical Rayleigh number is in the range of 14000–16000. The uncertainties in the salinity measurement constitute the major contribution to the uncertainties in R. It is estimated that the density gradient may be determined to within ± 3 per cent. Since it enters the Rayleigh number to the third power, the Rayleigh number as reported has an uncertainty of ± 10 per cent. In view of this fact, we estimate the critical Rayleigh number to be 15000 ± 2500.

Conclusions

1. We have shown experimentally that cellular convection induced by horizontal heating is an instability phenomena.
2. The length scale $h = \alpha \Delta T / \phi_0$ is a proper choice as is borne out by the experimental results.
3. The critical Rayleigh number for cellular motion has been determined to be 15000 ± 2500.

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STABILITÉ D'UNE CONVECTION THERMIQUE DANS UN GRADIENT DE SALINITÉ DÛ À UN CHAUFFAGE LATÉRAL

Résumé—Les recherches expérimentales ont été conduites afin d'étudier le comportement du champ dynamique induit par un chauffage latéral dans un fluide stable, stratifié, de gradient constant. Les expériences sont conduites dans un réservoir de dimensions: 12,7 × 20 × 29,8 cm. Deux parois opposées en aluminium sont maintenues à des températures différentes mais constantes tandis que les deux autres sont en plexiglas pour permettre l'observation du mouvement. Le paramètre actif est le nombre de Rayleigh basé sur l'echelle de longueuer qui est l'élévation potentielle d'un élément de fluide chauffé dans un environnement stratifié stable. A des nombres de Rayleigh sous-critiques, le fluide s'élève presque imperceptiblement, parallèlement à la plaque chauffée et le transport latéral de chaleur se fait entièrement par conduction. A des nombres de Rayleigh supercritiques l'écoulement devient instable et une convection cellulaire se forme, clairement visible par la méthode des ombres. Les cellules sont de dimensions approximativement égales et elles apparaissent simultanément tout le long de la paroi chauffée. Elles croissent dans la direction latérale et atteignent finalement la paroi opposée. Le nombre de Rayleigh critique au dessous duquel se produit la convection cellulaire a pour valeur expérimentale 15000 ± 2500.

STABILITÄT DER NATÜRLICHEN KONVEKTION IN EINER SALZLÖSUNG MIT KONZENTRATIONSGRADIENT INFOLGE SEITLICHER BEHEIZUNG

Zusammenfassung—In einer stabil geschichteten Flüssigkeit mit konstantem Gradienten wird das Verhalten einer durch seitliches Beheizen hervorgerufenen Strömung experimentell untersucht. Die Versuche wurden in einem Behälter mit den Massen 12,7 × 20 × 29,8 cm ausgeführt; zwei gegenüberliegende Wände waren aus Aluminium und wurden auf verschiedenen, aber konstanten Temperaturen gehalten, die beiden anderen Wände waren aus Plexiglas, um die Strömung beobachten zu können. Der massgebliche Parameter ist die Rayleighzahl, gebildet mit einer Länge, die der Steighöhe eines aufgeheizten Flüssigkeitselements in einer stabil geschichteten Umgebung entspricht. Bei unterkritischen Rayleighzahlen steigt die Flüssigkeit fast unmerklich parallel zur beheizten Wand hoch, der horizontale Wärmetransport geschieht ausschliesslich durch Leitung. Bei überkritischen Rayleighzahlen wird die Strömung instabil und zellenförmige Konvektion setzt ein, die in der Schattenaufnahme deutlich sichtbar ist. Die Zellen sind von etwa gleicher Grösse und treten gleichzeitig entlang der beheizten Wand auf. Sie wachsen in horizontaler Richtung an und erreichen schliesslich die gegenüberliegende Wand. Die kritische Rayleighzahl, oberhalb der Konvektion auftritt, wurde experimentell zu 15000 ± 2500 bestimmt.

УСТОЙЧИВОСТЬ ТЕПЛОВОЙ КОНВЕКЦИИ ПРИ БОКОВОМ НАГРЕВЕ

Аннотация—Проведено экспериментальное исследование поведения поля течения потока, создаваемого боковым нагревом в стабильно расслоенной жидкости с постоянным градиентом. Эксперименты проводились в баке $5'' \times 7\frac{7}{8}'' \times 11\frac{3}{4}''$ высотой. Две противоположные стенки были сделаны из алюминия и имели различные но постоянные температуры, тогда как две другие были сделаны из плексигласа для визуальных наблюдений. Показано, что основным параметром является число Релея, основанное на линейном масштабе, который является потенциальным подьемом нагретого элемента жидкости в стабильно расслоенной среде. При докритических числах Релея жидкость поднимается почти незаметно параллельно нагретой пластине. Поперечный перенос тепла происходит исключительно путем теплопроводности. При сверхкритических числах Релея течение становится неустойчивым и возникает ячеистая конвекция, что отчетливо видно на экране. Размеры ячеек приблизительно одинаковы и появляются они одновременно вдоль всей нагретой стенки. Они растут в поперечном направлении и, наконец, достигают противоположной стенки. Экспериментально установлено, что критическое число Релея при превышении которого происходит ячеистая конвекция, равно 15000 ± 2500.

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